31.b) Find a context-free grammar G which generates the language L that consists of all the words in a and b with twice as many a's as b's.

32.a) i) Define a finite state automaton with an example.
   ii) Design an FSM that performs serial binary addition

(OR)

32.b) Find the DFA equivalent to the NFA for which the state table is given in table and the accepting state is S_2.

I S	F	
1.	a	b
S_0	S_0, S_1	S_2
S_1	S_0	S_1
S_2	S_1	S_0, S_1

PART-B   (5X4 = 20)
(Answer any five questions)

21. Show that the intersection of two subgroups of a group is a subgroup.

22. Find the code words generated by the encoding function $e : B^3 \to B^5$ with respect to parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

23. State and prove Handshaking theorem.

24. Find a Hamiltonian path or a Hamiltonian circuit, if it exists in each of the following two graphs. If it does not exist, explain why?
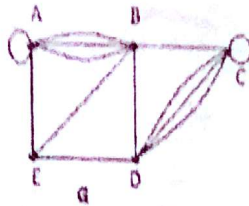

(G₁)   (G₂)

25. Find the number of vertices, the number of edges and the degree of each vertex in the following undirected graphs. Verify also the handshaking theorem.

26. Define the phrase structure grammar.

27. If $G = \{(S, A),(a, b), S, P\}$ where
$P = \{S \to aAS, S \to a, A \to SbA, A \to SS, A \to ba\}$, generate the string *aabbaa* by using (i) a left most derivation, (ii) a right most derivation.

PART-C

(5x12=60)

28.a) Prove that the necessary and sufficient conditions for a non-empty subset H of a group$(G, *)$ to be a subgroup is $a, b \in H \Rightarrow a * b^{-1} \in H$.

(OR)

28.b) Check whether $(R, *)$ defined by $a * b = a + b + 2ab$ is an abelian group or not?

29.a) Given the generator matrix G=
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

corresponding to the encoding function $e : B^4 \to B^7$. Find the corresponding parity check matrix and use it to decode the following received words and hence to find the original message.

29.b) Prove that every subgroup of a cyclic group is cyclic.

30.a) Find the shortest distance matrix and the corresponding shortest path matrix for all the pairs of vertices in the directed graph given in figure using Warshall's algorithm.
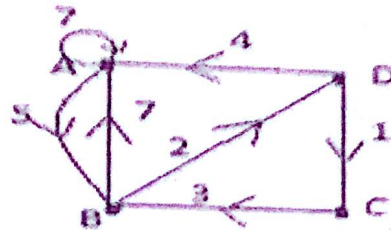
(OR)

30.b)Use Krushkal's algorithm to find a minimum spanning tree for the weighted graph shown in figure.
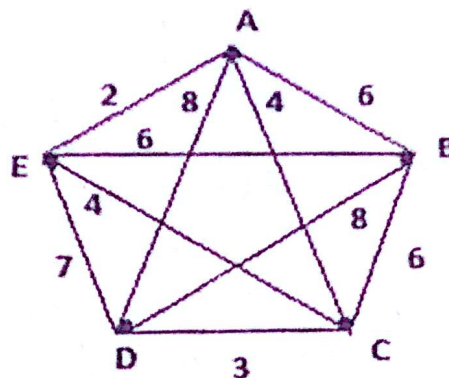
31.a). i) Prove that the number of edges in a bipartite graph with 'n' vertices is atmost $\left(\dfrac{n^2}{4}\right)$.

ii) Draw all the sub-graphs of K_3 containing atleast one vertex.

(OR)

SET-D

SRM UNIVERSITY
DEPARTMENT OF MATHEMATICS
CYCLE TEST-II
15MA203-Discrete Mathematics for IT

Total Marks: 20

Duration: 45 mins

PART-A   (20 X
1 = 20)
(Answer all questions)

1. In a cyclic group $G = \{ 1, -1, i, -i \}$ where $*$ is the usual multiplication the generators are ....
   a. 1, -1   b. 1, i   c. -1, -i   d. i, -i

2. Every subgroup of a cyclic group is
   a. Cyclic   b. Abelian c. Not cyclic   d. Not abelian.

3. The Minimum distance of a code $\{10110, 11110, 10011\}$ is
   a. 1   b. 2   c. 3   d. 4

4. In any group $(G, *)$, $(a * b)^{-1}$ is
   a. $a^{-1} * b^{-1}$ b. $b^{-1} * a^{-1}$ c. $a^{-1} * b$ d. $a * b^{-1}$

5. In any group $(G, *)$ if $a * a = a$ then
   a. $a = a^{-1}$ b. $a = e$ c. $a * b = b * a$ d. $a^2 = b^2$.

6. Let $(G, *)$ be a group, let a, b $\square$ G, If $b = a^n$, for all b $\square$ G, then $(G, *)$ is a
   a. Cyclic group with generator a
   b. Cyclic group with generator b
   c. Cyclic group with generator $(a * b)$
   d. Cyclic group with generator $(a*b)^{-1}$

7. If $x== 11010$ $y = 10101$ hen $H(x, y) =$
   a. 1   b. 2   c. 3   d. 4

8. A.............. is a finite altenating sequence of vertices and edges beginning and ending with the vertices such that each edge is incident on the vertex preceeding and following it.
   a. Simple Path   b. Cycle   c. Simple cycle   d. Path

9. A tree with n vertices has ....edges

    a.nC_2 b. nP_2 c. n-1   d. n!

10. A vertex with zero in degree is called ....
    a. Sink   b. Source c. Terminal   d. Out degree

11. The number of edges in a complete graph with n vertices is
....

    a. $\frac{n(n-1)}{2}$ b. $\frac{n}{2}$ c. $\frac{n!}{2}$ d. $\frac{n(n+1)(2n+1)}{2}$
                                              5.

12. A graph in which loops and parallel edges are allowed is called
.......grpah
    a. Pseoudo   b. Multi   c. Simple d. Null

13. A circuit of a graph G is called ... circuit if it includes each edge of G exactly once
    a.Hamiltonian b. Konisberg   c. Closed d. Eulerian

14. Name the grammar G in which a word in L(G) can be generated by more than one derivation of the same type
    a.CFG   b. CSG   c. Ambiguous   d. Unambiguous

15. Name the grammar G in which a word in L(G) can be generated by a unique derivation
    a. Regular b. PSG   c. ambiguous   d.unambiguous

16. Backus-Naur form is an alternative form
    a. PSG   b.regular c. ambiguous   d.CFG

17. Transition function in NFA assigns ..... next state to every pair of state and input
    a. all b. Unique c. two   d.several

18. Find the type of grammar which consists of the productions
S→aB, B→bB, B→bA, A→a, B→b}
    a. Type -0 b. Type -1 c. Type-2 d. Type -- 3

19. Find a language generated by athe grammar G={(S), (0, 1), S, P} with
P = {S→11S, S→0}
    a. $\{1^n 0\}$, n≥0 b. $\{1^{2n} 0\}$, n≥0 c. $\{1^n 0\}$, n≥1   d. $\{1^{2n} 0\}$, n≥1

20. The Language accepted by a finite automatan is called
    a. Regular language b. Context Free Language
    c. Context sensitive Language   d. Formal Language